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# Thermodynamics of $(1+1)$ dilatonic black holes in global flat embedding scheme

Soon-Tae Hong

*Department of Science Education, Ewha Womans University, Seoul 120-750, South Korea*

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**Abstract**

We study thermodynamics of $(1+1)$-dimensional dilatonic black holes in global embedding Minkowski space scheme. Exploiting geometrical entropy correction we construct consistent entropy for the charged dilatonic black hole. Moreover, $(1+1)$ dilatonic black holes with higher order terms are shown to possess $(3+2)$ global flat embedding structures regardless of the details of the lapse function, and to yield a generic entropy.


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Since $(1+1)$-dimensional black holes associated with string theory was proposed [1], there have been lots of progresses such as discovery of $U$-duality between two-dimensional dilatonic black holes [2–5] and five-dimensional one in the string theory. A thermal Hawking effect on a curved manifold [6,7] can be looked at as an Unruh effect [8] in a global embedding Minkowski space (GEMS). This GEMS approach [9–11] could suggest a unified derivation of thermodynamics for various curved manifolds [9] and the $(5+1)$ GEMS structure of $(3+1)$ Schwarzschild black hole solution [12] was obtained [9].

In this Letter we study thermodynamics of $(1+1)$ dilatonic black holes in the GEMS scheme. Using geometrical entropy correction we can obtain consistent entropy for a charged dilatonic black hole. More general $(1+1)$ dilatonic black holes are shown to possess $(3+2)$ GEMS structures regardless of the details of the lapse function with higher order terms, and to yield a generic entropy formula.

We start with two-dimensional dilatonic black holes [2–5] associated with the type IIA string theory and its compactification to five dimensions whose metric is the product of a three-sphere and an as-

*E-mail address:* soonhong@ewha.ac.kr (S.-T. Hong).


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ymptotically flat two-dimensional geometry. The ten-dimensional type IIA superstring solution consists of a solitonic NS 5-brane wrapping around the compact coordinates, say, $x_5$, $x_i$ $(i = 6, 7, 8, 9)$ and a fundamental string wrapping around $x_5$, and a gravitational wave propagating along $x_5$. In the string frame, the 10-metric, dilaton and 2-form field $B$ are given as [13–16]

$$\begin{aligned}
ds^2 &= -(H_1 K)^{-1} f\,dt^2 \\
&\quad + H_1^{-1} K\bigl(dx_5 - \bigl(K'^{-1} - 1\bigr)\,dt\bigr)^2 \\
&\quad + H_5\bigl(f^{-1}\,dr^2 + r^2\,d\Omega_3^2\bigr) + dx_i\,dx^i, \\
e^{-2\phi} &= H_1 H_5^{-1}, \\
B_{05} &= H_1'^{-1} - 1 + \tanh\alpha, \\
B_{056789} &= H_5'^{-1} - 1 + \tanh\beta,
\end{aligned}$$

where $r^2 = x_1^2 + \cdots + x_4^2$, $f = 1 - \frac{r_0^2}{r^2}$ and

$$\begin{aligned}
H_1 &= 1 + \frac{r_0^2 \sinh^2\alpha}{r^2}, \qquad H_5 = 1 + \frac{r_0^2 \sinh^2\beta}{r^2}, \\
K &= 1 + \frac{r_0^2 \sinh^2\gamma}{r^2}, \\
H_1'^{-1} &= 1 - \frac{r_0^2 \sinh\alpha\cosh\alpha}{r^2 H_1}, \\
K'^{-1} &= 1 - \frac{r_0^2 \sinh\gamma\cosh\gamma}{r^2 K}.
\end{aligned}$$

Here $B_{05}$ component of the Neveu–Schwarz 2-form $B$ is the electric field of fundamental string and $B_{056789}$ is the electric field dual to the magnetic field of the 5-brane with components $B_{ij}$. Exploiting dimensional reduction in the $x_5$, $x_i$ $(i = 6, 7, 8, 9)$ directions in the Einstein frame [13,14], and then performing an $T_5 S T_{6789} S T_5$ transformation [17] and an $\mathrm{SL}(2, R)$ coordinate transformation associated with the $\mathrm{O}(2, 2)$ $T$-duality group, together with the same set of reverse $S$ and $T$ transformations, one can obtain the five-dimensional black hole metric

$$\begin{aligned}
ds^2 &= -\bigl(H_1^{-3}\bar H_5\bigr)^{-1/4} K^{-1} f\,dt^2 \\
&\quad + \bigl(H_1^{-3}\bar H_5\bigr)^{-1/4} K\bigl(\bigl(dx_5 - K'^{-1} - 1\bigr)\,dt\bigr)^2 \\
&\quad + \bigl(H_1 \bar H_5^3\bigr)^{1/4}\bigl(f^{-1}\,dr^2 + r^2\,d\Omega_3^2\bigr) \\
&\quad + \bigl(H_1 \bar H_5^{-1}\bigr)^{1/4}\,dx_i\,dx^i,
\end{aligned} \tag{1}$$

$$e^{-2\phi} = \frac{r^2}{r_0^2} + \sinh^2\alpha, \tag{2}$$

where $\bar H_5 = r_0^2/r^2$. Next, performing dimensional reduction in the $x_5$, $x_i$ $(i = 6, 7, 8, 9)$ directions with $\alpha = \gamma$, one can arrive at the five-dimensional black hole metric [18]

$$\begin{aligned}
ds^2 &= -\left(1 - \frac{r_0^2}{r^2}\right)\left(1 + \frac{r_0^2 \sinh^2\alpha}{r^2}\right)^{-2} dt^2 \\
&\quad + \left(\frac{r^2}{r_0^2} - 1\right)^{-1} dr^2 + r_0^2\,d\Omega_3^2,
\end{aligned} \tag{3}$$

and the dilaton which is trivially invariant under the dimensional reduction to yield the above result (2). Here one notes that the metric (3) is the product of the two completely decoupled parts, namely, a three-sphere and an asymptotically flat two-dimensional geometry which describes the two-dimensional charged dilatonic black hole. Introducing a new variable $x$ with $Q = 2/r_0$

$$e^{Qx} = 2\left(\frac{r^2}{r_0^2} + \sinh^2\alpha\right)\bigl(m^2 - q^2\bigr)^{1/2},$$

where $m$ and $q$ are the mass and charge of the dilatonic black hole, one can obtain the well-known $(1 + 1)$ charged dilatonic black hole [2–4]

$$ds^2 = -N^2\,dt^2 + N^{-2}\,dx^2, \tag{4}$$

where the lapse function is given as

$$N^2 = 1 - 2me^{-Qx} + q^2 e^{-2Qx}.$$

We can then obtain the horizon $x_H$ and $x_-$ in terms of the mass $m$ and the charge $q$:

$$\begin{aligned}
e^{Qx_H} &= m + \bigl(m^2 - q^2\bigr)^{1/2}, \\
e^{Qx_-} &= m - \bigl(m^2 - q^2\bigr)^{1/2}.
\end{aligned} \tag{5}$$

By using these relations, we can rewrite the lapse function as

$$N^2 = \bigl(1 - e^{-Q(x - x_H)}\bigr)\bigl(1 - e^{-Q(x - x_-)}\bigr).$$

First, we consider the uncharged dilatonic black hole 2-metric

$$ds^2 = -\bigl(1 - 2me^{-Qx}\bigr)\,dt^2 + \bigl(1 - 2me^{-Qx}\bigr)^{-1}\,dx^2,$$

from which we can construct $(3 + 1)$-dimensional GEMS

$$z^0 = k_H^{-1}\bigl(1 - e^{-Q(x - x_H)}\bigr)^{1/2} \sinh k_H t,$$

$$
\begin{aligned}
z^1 &= k_H^{-1}\left(1-e^{-Q(x-x_H)}\right)^{1/2}\cosh k_H t,\\
z^2 &= x,\\
z^3 &= \frac{2}{Q}e^{-Q(x-x_H)/2},
\end{aligned}
\tag{6}
$$

where the surface gravity is given by $k_H = Q/2$. Using the GEMS (6) and the relation $G_4 = G_2 V_2$ (details of which will be discussed later), where $V_2$ is a compact volume $V_2 = 2/Q$ given along $z^2$ only, we can then obtain the desired entropy

$$
\begin{aligned}
S &= \frac{1}{4G_4}\int dz^2\,dz^3\,\delta\left(z^3 - \frac{2}{Q}e^{-Q(z^2-x_H)/2}\right)\\
&= \frac{1}{4G_2},
\end{aligned}
\tag{7}
$$

which is consistent with the previous result in [3,18].

Second, for a charged dilatonic black hole case associated with the metric (4), we can construct a $(3+2)$ GEMS $ds^2 = -(dz^0)^2 + (dz^1)^2 + (dz^2)^2 + (dz^3)^2 - (dz^4)^2$ given by the coordinate transformations,

$$
\begin{aligned}
z^0 &= k_H^{-1}\left(1-e^{-Q(x-x_H)}\right)^{1/2}\left(1-e^{-Q(x-x_-)}\right)^{1/2}\\
&\quad\times \sinh k_H t,\\
z^1 &= k_H^{-1}\left(1-e^{-Q(x-x_H)}\right)^{1/2}\left(1-e^{-Q(x-x_-)}\right)^{1/2}\\
&\quad\times \cosh k_H t,\\
z^2 &= x,\\
z^3 &= \frac{2}{Q}\left(1+e^{Q(x_H-x_-)}\right)^{1/2}\sin^{-1} e^{-Q(x-x_-)/2}\\
&\equiv f\left(z^2\right),\\
z^4 &= \frac{2e^{-3Q(x-x_H)/2}e^{-Q(x-x_-)/2}}{Q(e^{-Q(x-x_H)}-e^{-Q(x-x_-)})} \equiv g\left(z^2\right),
\end{aligned}
\tag{8}
$$

where the surface gravity is given by

$$
k_H = \frac{Q}{2}\left(1-e^{-Q(x_H-x_-)}\right).
$$

Here one can also check that, in the uncharged limit $q \to 0$, the above coordinate transformations are exactly reduced to the previous one (6) for the uncharged dilatonic black hole case. Moreover, one can easily obtain the relation between $z^3$ and $z^4$ as follows

$$
\begin{aligned}
z^4 &= \frac{2e^{3Q(x_H-x_-)/2}}{Q(e^{Q(x_H-x_-)}-1)}\sin^2\left[\frac{Qz^3}{2(1+e^{Q(x_H-x_-)})^{1/2}}\right]\\
&\equiv h\left(z^3\right).
\end{aligned}
\tag{9}
$$

In the standard GEMS approach, all the informations for the entropy come from the areas themselves associated with the event horizons. Here the Newton constants $G_n$ in the higher-dimensional embeddings are implicitly treated to be the same as the original $G_d$ of the $d$-dimensional black holes [9]. However, in the $(1+1)$ dilatonic black hole cases, we could not obtain the areas in terms of the event horizons due to the delta-function-like behaviors at the event horizons, which are characteristics of the $(1+1)$ dilatonic black holes. As in (7), in order to obtain the entropies, we thus exploit an alternative scheme, where the entropy informations are extracted from the Newton constants $G_n$ which are now splitted into two factors: $G_n = G_d \times V_{n-d}$ with the volumes of the compact manifolds $V_{n-d}$. To be more specific, in order to calculate the entropy for the charged dilatonic black hole, we first consider a detector on the event horizon at $x = x_H$ where the detector only sees a compact manifold $V_3$ along the $z_3$ and $z_4$ directions, given by

$$
\begin{aligned}
V_3(x_H) &= \int dz^2\,dz^3\,dz^4\,\delta\left(z^3 - f\left(z^2\right)\right)\delta\left(z^4 - f\left(z^3\right)\right)\\
&= z^4(x_H).
\end{aligned}
$$

The Newton constant is then given by $G_5 = G_2 V_3(x_H)$ to yield the entropy at $x = x_H$

$$
\begin{aligned}
S(x_H) &= \frac{1}{4G_5}\int dz^2\,dz^3\,dz^4\,\delta\left(z^3 - f\left(z^2\right)\right)\\
&\quad\times \delta\left(z^4 - h\left(z^3\right)\right)\\
&= \frac{1}{4G_2}.
\end{aligned}
\tag{10}
$$

Note that, even though we have used the $x_H(q)$ in calculation of the above entropy $S(x_H)$, the final result does not contain any information of the charge $q$ and mass $m$ associated with the even horizons $x_H$ and $x_-$, to yield the same entropy (7) of the uncharged case.

Different from the uncharged case, we have another event horizon $x = x_-$, where we have another compact manifold with volume $V_3(x_-) = z^4(x_-)$ to yield the modified Newton constant $\tilde{G}_5 = G_2 \tilde{V}_3$ with $\tilde{V}_3 = V_3(x_H) + V_3(x_-) = z^4(x_H) + z^4(x_-)$, since the detector at the event horizon $x = x_-$ can see two compact manifolds at $x = x_-$ and $x = x_H$. Moreover, it has been claimed in [19] that the entropy of a charged black hole should decrease with the absolute value of the black hole charge. We can then obtain the entropy loss due to the existence of the compact manifold at

$x = x_-$

$$\delta S = \frac{1}{4\tilde{G}_5}\int dz^2\,dz^3\,dz^4\,\delta\big(z^3 - f\big(z^2\big)\big)\delta\big(z^4 - h\big(z^3\big)\big)$$
$$= \frac{1}{4G_2}\frac{z^4(x_H)}{z^4(x_H) + z^4(x_-)},$$

to yield the total entropy $S = S(x_H) - \delta S$ of the dilatonic charged black hole as follows

$$S = \frac{1}{4G_2}\frac{z^4(x_-)}{z^4(x_-) + z^4(x_-)}$$
$$= \frac{1}{4G_2}\frac{m + (m^2 - q^2)^{1/2}}{2m}, \tag{11}$$

which is consistent with the previous result in [3,18]. Note that in the vanishing charge limit $q \to 0$, the above entropy is reduced to that of uncharged case (7). Moreover, without the $U$-duality transformations discussed above, we can obtain the consistent entropy (11) via the GEMS embeddings and their associated geometrical entropy corrections.

Following the standard procedure in general relativity, one can obtain the 2-acceleration, the Hawking temperature and the black hole temperature

$$a_2 = \frac{Qe^{-Qx}(m - q^2 e^{-Qx})}{(1 - e^{-2Q(x-x_H)})^{1/2}(1 - e^{-2Q(x-x_-)})^{1/2}},$$
$$T_H = \frac{a_5}{2\pi}$$
$$= \frac{Q}{4\pi}\frac{1 - e^{-Q(x_H - x_-)}}{(1 - e^{-Q(x-x_H)})^{1/2}(1 - e^{-Q(x-x_-)})^{1/2}},$$
$$T = NT_H = \frac{Q}{4\pi}\big(1 - e^{-Q(x_H - x_-)}\big),$$

where we have used the Killing vector $\xi = \partial_t$ on the two-dimensional dilatonic charged black hole manifold described by $(t, x)$ for the trajectories. Here note that the above Hawking temperature $T_H$ is also given by the relation in (13) [6].

Next, we consider more general dilatonic black holes associated with the on-shell action [3,4]

$$I = \int d^2x\,\big[-4\nabla^a\big(e^{-2\phi}\nabla_a\phi\big) + e^{-2\phi}\big(R + 2\nabla^2\phi\big)\big],$$

where the dilaton field is given by $\phi(x) = \phi_0 - \frac{1}{2}Qx$ and the 2-metric is given by (4) with the lapse function

$$N^2 = 1 + \sum_{n=1} c_n e^{-nQx}, \tag{12}$$

where $c_1 = -2m$, $c_2 = q^2$ and $c_n$ $(n \geqslant 3)$ are coefficients of higher order terms. Note that the lapse function (12) can be rewritten in terms of the event horizons $x_n$ $(n = 1, 2, \ldots)$ with $x_1 = x_H$ and $x_n > x_{n+1}$,

$$N^2 = \prod_{n=1}\big(1 - e^{-Q(x-x_n)}\big).$$

As in the previous case, we can obtain the surface gravity, the 2-acceleration and the Hawking temperature in more general form

$$k_H = N\frac{dN}{dx}\bigg|_{x=x_H}, \qquad a_2 = \frac{dN}{dx},$$
$$T_H = \frac{1}{2\pi}\frac{k_H}{N}, \tag{13}$$

which are independent of the dimensionality of the GEMS structures. We construct the GEMS embedding solutions for our general $(1+1)$ dilatonic black hole by making an ansatz of three coordinates $(z^0, z^1, z^2)$ in (15) to yield

$$-\big(dz^0\big)^2 + \big(dz^1\big)^2 + \big(dz^2\big)^2$$
$$= ds^2 - \left(N^{-2} - k_H^{-2}\left(\frac{dN}{dx}\right)^2 - 1\right)dx^2$$
$$\equiv ds^2 - \big(dz^3\big)^2 + \big(dz^4\big)^2.$$

Here we have used the fact that the terms in the parenthesis in the second line can be expressed in terms of difference of two positive definite terms

$$N^{-2} - k_H^{-2}\left(\frac{dN}{dx}\right)^2 - 1 \equiv F^2 - G^2, \tag{14}$$

where $F$ and $G$ can be read off from (15). We can thus obtain the $(3+2)$-dimensional GEMS $ds^2 = -(dz^0)^2 + (dz^1)^2 + (dz^2)^2 + (dz^3)^2 - (dz^4)^2$ given by the coordinate transformations,

$$z^0 = k_H^{-1}N\sinh k_H t,$$
$$z^1 = k_H^{-1}N\cosh k_H t,$$
$$z^2 = x,$$
$$z_3 = \int dx\,F(x) \equiv f\big(z^2\big),$$
$$z^4 = \int dx\,G(x) \equiv g\big(z^2\big). \tag{15}$$

Note that, as in (9), $z^4$ can be expressed in terms of $z^3$: $z^4 = g \cdot f^{-1}(z^3) \equiv h(z^3)$.

Now we comment on the dimensionality of the GEMS embeddings in the general dilatonic black holes. The charge parameter $c_2 = q^2$ introduces one more time-like dimension to yield two time dimensionalities with $(3 + 2)$ GEMS structures for the charged dilatonic black hole. In the general dilatonic black holes with $c_n$ $(n = 1, 2, \ldots)$, even though we have horizons $x_n$ more than two ones $x_H$ and $x_-$ of the charged dilatonic black hole, the GEMS structures are fixed as $(3 + 2)$ dimensions with no more increasing dimensionality, since only two positive definite terms $F^2$ and $G^2$ are enough to describe the terms of (14) regardless of whatever the lapse function $N^2$ has higher order terms with $c_n$ $(n = 1, 2, \ldots)$.

Next, we calculate the entropy for the general $(1 + 1)$ dilatonic black hole with higher order terms. As in the charged dilatonic black hole case, a detector on the event horizon at $x = x_H$ only sees a compact manifold $V_3$ along the $z_3$ and $z_4$ directions to yield the entropy (10) at $x = x_H$. However, we have other event horizons $x = x_n$ $(n = 2, 3, \ldots)$ associated with compact manifolds with volumes $V_3(x_n) = z^4(x_n)$ to yield the Newton constant

$$\tilde{G}_5 = G_2 \sum_{n=1} V_3(x_n) = G_2 \sum_{n=1} z^4(x_n).$$

The existences of the compact manifolds at $x = x_n$ $(n = 2, 3, \ldots)$ thus yield the geometrical entropy correction originated from $\tilde{G}_5$

$$\delta S = \frac{1}{4G_2} \frac{z^4(x_H)}{\sum_{n=1} z^4(x_n)},$$

so that, together with $S(x_H)$ which has the same form as (10), we can obtain the total entropy $S = S(x_H) - \delta S$ of the general $(1 + 1)$ dilatonic black hole

$$S = \frac{1}{4G_2} \frac{\sum_{n=2} z^4(x_n)}{\sum_{n=1} z^4(x_n)}. \tag{16}$$

In the charged case with $c_1 = -2m$, $c_2 = q^2$ and $c_n = 0$ $(n \geqslant 3)$, by exploiting the explicit expression for $z^4$ in the GEMS (8) we obtain for the horizons $x_1 = x_H$ and $x_2 = x_-$

$$z^4(x_1) = \frac{2e^{Q(x_1+x_2)/2}}{Q(e^{Qx_1} - e^{Qx_2})},$$
$$z^4(x_2) = \frac{2e^{Q(3x_1-x_2)/2}}{Q(e^{Qx_1} - e^{Qx_2})}. \tag{17}$$

After some algebra with the identities (5), substitution of $z^4(x_1)$ and $z^4(x_2)$ in (17) into the generic entropy formula (16) reproduces the previous result (11). Similarly, for the uncharged case with $c_1 = -2m$ and $c_n = 0$ $(n \geqslant 2)$, we can easily check that the entropy (16) is reduced to the previous one (7). For more general cases with $c_1 = -2m$, $c_2 = q^2$ and nonvanishing $c_n$ $(n \geqslant 3)$, we can find the expression for $z^4$ in the GEMS (15), which is given by an integral form. Different from the charged case with $z^4(x_n)$ $(n = 1, 2)$ in (17), for this general dilatonic black hole we do not have explicit analytic expressions for $z^4(x_n)$ at the moment so that we cannot proceed to evaluate the entropy via the formula (16). However, if the coefficients $c_n$ are given explicitly, we can find $x_n$ and $z^4(x_n)$, with which the generic entropy (16) is supposed to yield to all order a result consistent with that given in [3].

In conclusion, we have investigated the higher-dimensional global flat embeddings of $(1 + 1)$ (un)charged and general dilatonic black holes. These two-dimensional dilatonic black holes are shown to be embedded in the $(3 + 1)$ and $(3 + 2)$ dimensions for the uncharged and charged two-dimensional dilatonic black holes, respectively. Moreover, in the general dilatonic black holes with higher order terms, even though we have horizons $x_n$ more than two ones $x_H$ and $x_-$ of the charged dilatonic black hole, the GEMS structures have been shown to be fixed as $(3 + 2)$ with no more increasing dimensionality.

Different from the uncharged case, in order to obtain the entropy of the $(1 + 1)$ charged dilatonic black holes, we have taken into account all the compact manifold associated with the event horizons to yield the modified Newton constant. Exploiting the geometrical entropy correction originated from the modified Newton constant, we have obtained the entropy for the charged dilatonic black holes and even for the general dilatonic black holes. It is quite significant to obtain the consistent entropies through the GEMS embeddings and their associated geometrical entropy corrections, without getting involved in the $U$-duality transformations associated with the type IIA string theory.

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